Turing's Economics
A Birth Centennial *Homage*♣

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"Quantum mechanists always seem to require infinitely many dimensions; I don't think I can cope with so many – I'm going to have about 100 or so – that ought to be enough don't you think?"

Alan Turing to Robin Gandy♠

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♣ An invited contribution to the **OECD 'blog'** (OECD Insights), to commemorate the *Alan Turing Birth Centennial*. I am deeply indebted to Dr. Patrick Love for inviting me to contribute this *homage* to one of 20^{th} century's most imaginative scientists, whose works have influenced the way I have approached formalism in economic theory, behavioural economics and economic dynamics.

♥ **A**lgorithmic **S**ocial **S**ciences **R**esearch **U**nit.

♠ **Collected Works of A.M. Turing – Mathematical Logic**, edited by R. O. Gandy & C.E.M. Yates North-Holland, Amsterdam, p. 266.

♦ Strangely, in his personal memoir, *My Brother Alan*, John Turing states: 'My brother Alan was born on 21 June 1912 in a London Nursing Home' (p.145, John Turing, 2012).

It is little realised that what I call the *Five Turing Classics – On Computable Numbers* (Turing 1936-7), *Systems of Logic* (Turing, 1939), *Computing Machinery and Intelligence* (Turing, 1950), *The Chemical Basis of Morphogenesis* (1952) and *Solvable and Unsolvable Problems* (1954) – should be read together to understand why there can be something called *Turing's Economics*[1].

Herbert Simon, as one of the acknowledged founding fathers of *computational cognitive science* was deeply indebted to Turing in the way he tried to fashion what I have called *Computable Economics* (Velupillai, 2000)[2]. It was not for nothing that Simon warmly acknowledged, in his essay in a volume 'memorializing Turing' (Simon, 1996, p. 81):

> "If we hurry, we can catch up to Turing on the path he pointed out to us so many years ago."

Simon was on that path, for almost the whole of his research life. It is my hope that an economics of the future would follow Simon, and 'catch up to Turing on the path he pointed out to us so many years ago'. It has been my mission, first to learn to take this 'path', and then to teach others the excitement and fertility, for economic research, of taking the path Turing 'pointed out to us so many years ago'.

A comparison of Turing's classic formulation of *Solvable and Unsolvable Problems* and Simon's variation on that theme, as *Human Problem Solving* (Newell & Simon, 1972), would show that the *human problem solver* in the world of Simon needs to be defined – as Simon did - in the same way Turing's approach to *Solvable and Unsolvable Problems* was built on the foundations he had established in his classic of 1936-37.

At a deeper epistemological level, I have come to characterize the distinction between orthodox economic theory and Turing's Economics in terms of the last sentence of the last published paper by Alan Turing (1954, p. 23; italics added):

> "These, and some other results of mathematical logic may be regarded as going some way towards a demonstration, within mathematics itself, of the *inadequacy of 'reason' unsupported by common sense*."

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[1] It is interesting to note that the five contributions came in two clusters, the first two in 1936-7 & 1938/9; the last three in the fertile last four years of his tragically brief life.

[2] I could as well have called it *Turing's Economics*.

We – at ASSRU – characterize every kind of orthodox economic theory, including orthodox behavioural economics[3], advocating the *adequacy* of 'reason' unsupported by common sense; contrariwise, in Turing's economics we take seriously what we now refer to as *Turing's Precept*: 'the inadequacy of reason unsupported by common sense'.

At another frontier of research in many of what are fashionably referred to as 'the sciences of complexity', some references to Turing's classic on *The Chemical Basis of Morphogenesis* is becoming routine, even in varieties of computational economics[4] exercises, especially when concepts such as 'emergence' are invoked. Just as he had done in the case of *Solvable and Unsolvable Problems,* mulling over the nature and structure of seriously complex games such as Chess and GO, before interpreting the solvability of such games in terms of the mathematics of his 1936-7 classic on *Computable Numbers*, the contents of D'Arcy Thompson's classic, *On Growth and Form* (D'Arcy Thompson, [1917], 1942), preoccupied his fertile mind for over a decade and a half before the Morphogenesis classic came to fruition (see Hodges, 1983, pp. 207-8)[5].

It is now increasingly realized that the notion of 'emergence' originates in the works of the British Emergentists, from John Stuart Mill to C. Lloyd Morgan, in the half-century straddling the last quarter of the 19th and the first quarter of the 20th century. However, a premature Obituary of *British Emergentism* was proclaimed on the basis of a rare, rash, claim by Dirac:

> "The underlying physical laws necessary for the mathematical theory of a large part of physics and *the whole of chemistry are thus completely known*, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems *without too much computation*."
> Dirac (1929), p. 714; italics added.

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[3] Which I now refer to as *Modern Behavioural Economics*, to contrast it with the *Computably* underpinned *Classical Behavioural Economics* of Herbert Simon.

[4] Which has to be carefully distinguished from *Computable* – or, *Turing's – Economics*.

[5] It is a remarkable fact that Thom's classic on Structural Stability and Morphogenesis (Thom. 1975), equally inspired **by On Growth and Form**, and devoted to a dynamical systems approach to morphogenesis, as in Turing (1952), makes no reference whatsoever to the latter classic! I may be excused for adding one small personal note here. Around 1980, my Cambridge maestro, Richard Goodwin, also passionate about the dynamical systems interpretation of morphogenesis and dedicated to an interpretation of Schumpeterian 'creative destruction' in terms of it, asked me which of his books I wanted. With some embarrassment I indicated that I would wish to have his copies of Schumpeter's classics and the original edition of On Growth and Form. He warmly agreed to my 'request', and marked his copies of these books with a note that they were to go to me, whenever time's tenancy on his life ran out! Alas, his home was burgled shortly after his death and I never had the pleasure of inheriting his classics by Schumpeter and D'Arcy Thompson!

Contrast this with Turing's wonderfully laconic, yet eminently sensible precept (Turing, 1954, p. 9; italics added):

> "No mathematical method can be *useful* for *any* problem *if it involves much calculation*."

Turing's remarkably original work on *The Chemical Basis of Morphogenesis* was neither inspired by, nor influenced any later allegiance to the British Emergentist's tradition – such as the influential experimental and theoretical neurological and neurophilosophical work of Nobel Laureate, Roger Sperry[6].

On the other hand, the structure of the experimental framework Turing chose to construct was uncannily similar to the one devised by Fermi, Pasta and Ulam, (1955), although with different purposes in mind. But there was – and there remains – a deeper affinity in that the violation of the equipartition of energy principle that was observed in the Fermi-Pasta-Ulam simulation and the symmetry-breaking that is intrinsic to the dynamical system behaviour of Turing's system of reaction-diffusion equations.

Turing's aim was to devise a mechanism by which a spatially homogeneous distribution of chemicals – i.e., formless or patternless structure - could give rise to form or patterns via what has come to be called a *Turing Bifurcation*. A reaction-diffusion mechanism formalised as a (linear) dynamical system and subject to what I have referred to, in other writings, as *the linear mouse theory of self-organisation*[7].

In this same vein, it is most satisfying to note the role the *Turing Bifurcation* played in the development of the *Brusselator* and the work of the 1977 Chemistry Nobel Prize winner, Ilya Prigogine (cf. Nicoils and Prigogine, 1977) on self-organisation in non-equilibrium systems. Those seriously interested in the nonlinear, endogenous, theory of the business cycle, know

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[6] See, in particular, Sperry's outstanding *Noble Prize Lecture*, delivered on 8 December, 1981, on the nature of the emergence of consciousness and its relation to brain processing.

[7] In typically playful fashion, he summarised the mathematical mechanism he sought (Turing, 1952, pp. 43-4; italics added):

> "Unstable equilibrium is not … a condition which occurs very naturally. .. Since systems tend to leave unstable equilibria they cannot often be in them. *Such equilibria can, however, occur naturally through a stable equilibrium changing into an unstable one*. For example, if a rod is hanging from a point a little above its centre of gravity it will be in stable equilibrium. If, however, a mouse climbs up the rod the equilibrium eventually becomes unstable and the rod starts to swing. … The system which was originally discussed … might be supposed to correspond to the mouse somehow reaching the top of the pendulum without disaster, perhaps by falling vertically on to it."

very well that the *Turing Bifurctions* are at least as relevant as the *Hopf Bifurcation*, in modeling the 'emergence' and persistence of unstable dynamics, in aggregative economic dynamics.

Turing's Economics straddles the micro-macro divide in a way that makes the notion of microfoundations of macroeconomics thoroughly irrelevant; more importantly, it is also a way of circumventing the excessive claims of reductionists in economics, and their obverse! This paradox would have, I conjecture, provided much amusement to the mischievous child that Turing was, all his life.

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